# The Study of Solving the One-dimensional Infinite Square Well Quantum System by the Students in Department of Physics

Tzu Chyang King[1], Kuei Fang Tsai[2], Chen Ming Chen[3]*
department of applied physics, Pingtung University of Education, Pingtung
department of special education, Pingtung University of Education, Pingtung
department of special education, National Chiayi University, Chiayi
tomson2@ms18.hinet.net

***Abstract:*** *The first example for the undergraduate students enrolling quantum physics usually is the one-dimensional infinite square well system, which including most of the basic concepts of the quantum physics. This study analyses the test items' quality and the students' learning performance with S-P chart (Student-problem chart analysis theory). According to this analysis, teachers can diagnose students' learn performance and modify their teaching strategy.*
**Keywords:** quantum physics, ordering theory, S-P chart

## 1. Introduction

In Taiwan, modern physics are one of the major courses for junior students in Department of Physics, and such a curse is also provided in many other science and engineering departments. Its content usually includes the development of quantum mechanics, elementary quantum mechanics, special relativity, molecular physics, basic solid-state physics, atomic nuclei, elementary particles, and quantum statistics. The course with similar scope and content are also provided at colleges/universities in various countries around the world; however, the course names may vary. For example, modern physics is called "atomic physics" in China. Modern physics is different from classical physics. It describes the phenomena in the microscopic world and its concepts are abstract and incomprehensible. Moreover, because it usually involves complicated mathematics, many students experience difficulties in learning this course.

Students face great challenges in learning modern physics, especially quantum mechanics. Quantum mechanics is a physics theory describing microscopic substances and is the revolutionary paradigm for people to understand the structure of substances and their interactions. However, in the beginning of learning, most students are restricted by previous experiences and tend to learn it through their sensory perception and intuitive knowledge, while fail to correctly understand the concept of quantum mechanics. Such learning condition may pass the assessment of teachers, but the students fail to thoroughly understand the physical concept in the mathematical equations. For example, students tend to use classical mechanics in combination with the concept of quantum mechanics to explain the phenomenon of quantum mechanics (Fischler & Lichtfeldt, 1992), and thus develop "conflicting quantum thinking" or "conflicting mechanistic thinking" (Ireson, 2000). Gilbert and Boulter (1995) termed such hybrid concept the "hybrid models." In other words, such models fail to meet the concept of classicalmechanics or quantum mechanics.

To probe into the aforementioned issues, there is a need to understand the level of students' quantum mechanics concept. In the studies of scientific education, the prior knowledge of learners has different names, such as preconceptions, misconceptions, or alternative conceptions (Minstrell, 1982; Clement, 1983; McCloskey, 1983). In physics, prior knowledge is also called intuitive knowledge (p-prims) (Smith, diSessa & Roschelle, 1993; diSessa, 1993). Such knowledge exists in students' concepts and is hard to be changed.

Faced with students' difficulty in learning physics, many researchers attribute the problem to the mental sign, namely, mental model, constructed by students when they interact with the world (Gentner & Gentner, 1983; Johnson-Laird, 1983; Greca & Moreira, 2000). In the education of physics, the investigation on issues concerning mental model is mainly associated with issues of meaning learning (Ausubel, 1963). According to Ausubel's theory, the priority condition is to confirm students' known knowledge and to start to teach them from what they have known. According to diSessa (1993), mental model (e.g. concepts of physics) is at the high level of cognitive mechanism. The intuitive knowledge (p-prims) of small units is at the medium level, while the sensory schema is at the low level (e.g. sensory perception). Therefore, to assist students in overcoming the difficulties in learning physics, the priority should be given to understanding their comprehension of learning materials and the difficulties they encounter.

The first quantum mechanics problem discussed by most students in modern physics course is the status of a particle in a one-dimensional infinite square well. Such an example can be used as a good teaching material because it includes the major concepts of quantum mechanics and the mathematics involved is relatively simple. Therefore, students can easily solve the problem. If students can fully understand the meaning of various aspects of this problem, they can thoroughly understand the overall concepts of quantum mechanics. Unfortunately, most teachers do not pay particular attention to the importance of this example, and proceed to the next section after only explaining this example once. With years of experiences in the instruction of modern physics, the author suggests that it is necessary to guide students in reflecting on the abundant physical meanings hidden behind the example from different perspectives. If students can better understand this example and establish correct connection among various concepts, their subsequent learning is simply to repeatedly verify the conceptual framework, which will be beneficial to their learning.

Based on the aforesaid research background and motives, the purposes of this study are as follows: 1) to test the correlation between students' learning performance of the course and that in this example; 2) to investigate the problem solving performance of physics students for "one-dimensional infinite square well" in order to better understand their learning situation and difficulties in this example.

## 2. Research Methods

The data of students' problem solving performance for one-dimensional infinite square well were collected for analysis. The raw data were from the paper-based test. The researcher analyzed students' problem solving process and converted it into scoring criteria. The students' problem solving processes were converted into data according to the criteria for subsequent analysis. The research subjects, research tools, and data analyses are described as follows:

### *2.1 Research Subjects and Scope*

This study selected junior students taking the course of modern physics in a certain academic year at the school where the research served as the instructor. The effective sample was 79 students. In recent years, the admission performance of the students in the department has been approximately PR 50. In general, the willingness to learn is not high. The researcher investigated the daily learning hours of students, and found that, on average, student spent less than 1 hour on schoolwork.

### *2.2 Research Tools*

This study used two research tools. One of them was the "Infinite Square Well Problem Solving Concept Form" developed by the researcher according to the content of problem solving. The concepts required for the problem solving for the example where a single particle is in an infinite square swell were analyzed. The last row of the form showed the five stages of problem solving from the left to the right. Each stage was sub-divided into 1 to 3 items from the top to the bottom. An operational definition was developed for each item as the scoring criteria. The students meeting the criteria would be scored 1 point, and those failing to meet to criteria would be scored 0 point. After the draft chart was completed, two professors specializing in this field were invited to review, discuss about, and amend the form. The content of the finalized chart is shown in Table 1. This tool was used to convert students' responses into scores to facilitate the subsequent analysis.

The other research tool was an assessment test developed based on research tool 1. The test items included Q&A

questions and calculation questions. The test lasts for two hours.

Table 1. Infinite Square Well Problem Solving Concept Chart

| | | | | |
|---|---|---|---|---|
| | | C3Knowing the origin of boundary conditions. | | |
| | | C2 Using boundary conditions to calculate the energy levels and coefficients of differential equations. | D2 Being able to calculate the probability distribution of various energy states and to calculate the probability value of various energy states in various positions. | E2 Being able to use Heisenberg Uncertainty Principle to explain that when the system size is extremely large, energy states cannot be distinguished and the probability density is even. |
| A1 Writing down correctly that a particle's Schrodinger eqs. that will not change with time in various areas | B1Correctly converting Schrodinger eqs. in various areas into second order differential equations. | C1Writing down correctly the boundary conditions. | D1Writign down correctly the normalization conditions. | E1Being able to use Heisenberg Uncertainty Principle to explain that when the number of quantum is extremely large, the analysis result of quantum mechanics is consistent with that of classical mechanics and the probability density is even. |
| A:Understanding the system and principles of physics | B: Mathematical techniques for solving differential equations | C: Boundary conditions | D: Normalization conditions | E: Quantum mechanics is more inclined to classic mechanics |

### *2.3 Data Analysis*

This study used student-problem score chart (S-P chart) (Sato, 1980) as the analysis method. This method provided item caution index and student caution index. Therefore, the test items and individual student's problem solving and scoring pattern could be analyzed concurrently to effectively diagnose their difficulties in learning.

Moreover, to verify that this example is an important starting example of quantum mechanics, this study used Pearson's correlation coefficient to calculate the correlation between students' problem solving performance for this example and their academic performance.

## 3. Results and Discussion

In addition to being used to analyze the difference coefficient of the whole test, S-P chart can also be used to calculate the caution index of each test item and each student. The caution index of test items was abbreviated as CP, and that of students was abbreviated as CS. Such coefficients could be used to determine whether students' problem solving and scoring patterns were abnormal and whether "attention" was required. These data all could be used to assist teachers in diagnosing students' performance and test quality. In other words, they were effective tools for measuring teaching achievement and being provided as reference for improving teaching and test development and counseling students (You &, Yu, 2006). The analyses on the correlation among test items quality, students' problem solving performance and patterns, and students' problem solving performance and learning performance are described as follows.

### *3.1 Analysis on Test Items Quality*

Correct answer rate and caution index were used as the analysis indicators for test items quality. The caution index refers to the ratio of the difference between actual response pattern and perfect response pattern over maximum difference in the S-P chart. Sato (1980) suggested that item caution index should be used as the horizontal axis (cut-off point was.5) and the percentage of students providing the right answers should be used as the vertical axis (cut-off point was.5) to develop the test item diagnostic analysis chart where the properties of test items are divided into four categories. The analysis results are shown in Figure 1. The nine assessment items in this study are divided into two major categories. The first category is "Test items that are quite adequate," which could be used to distinguish the difference between low achievers and other students and are the more basic items in various problem-solving stages. The second category is "Test items that are difficult and suitable for distinguishing high achievers from other students." The quality of the test items is good, suggesting that the construct validity of such an assessment classification is acceptable.

As shown in Table 1, the Infinite Square Well Problem Solving Concept Chart, the five problem solving stages are from the left to the right, and the levels of each stage are from the top to the bottom. Figure 1 shows that, in the first three stages (A, B, and C), except for C, the correct answer rate is high. In the last two stages (D and E), except for D1, the correct answer rate is low. The results are reasonable because students would all consider forcing themselves to memorize the concepts when preparing for examinations and whether they understood such concepts is the second priority. As for C1 and D1, the correct answer rate of them is high because the test items are answered dependent on memory. Although the test item of C2 involves inference, the problem solving process could be memorized. Therefore, the correct answer rate is not low. The test item of C3 is the calculation basis. The thinking level is raised to comprehension. If students could not thoroughly understand the reason, the correct answer rate would be low. The test item of D2 is the calculation of the final result which involves students' computing ability. Because the computing ability is generally not high, the correct answer rate is low.

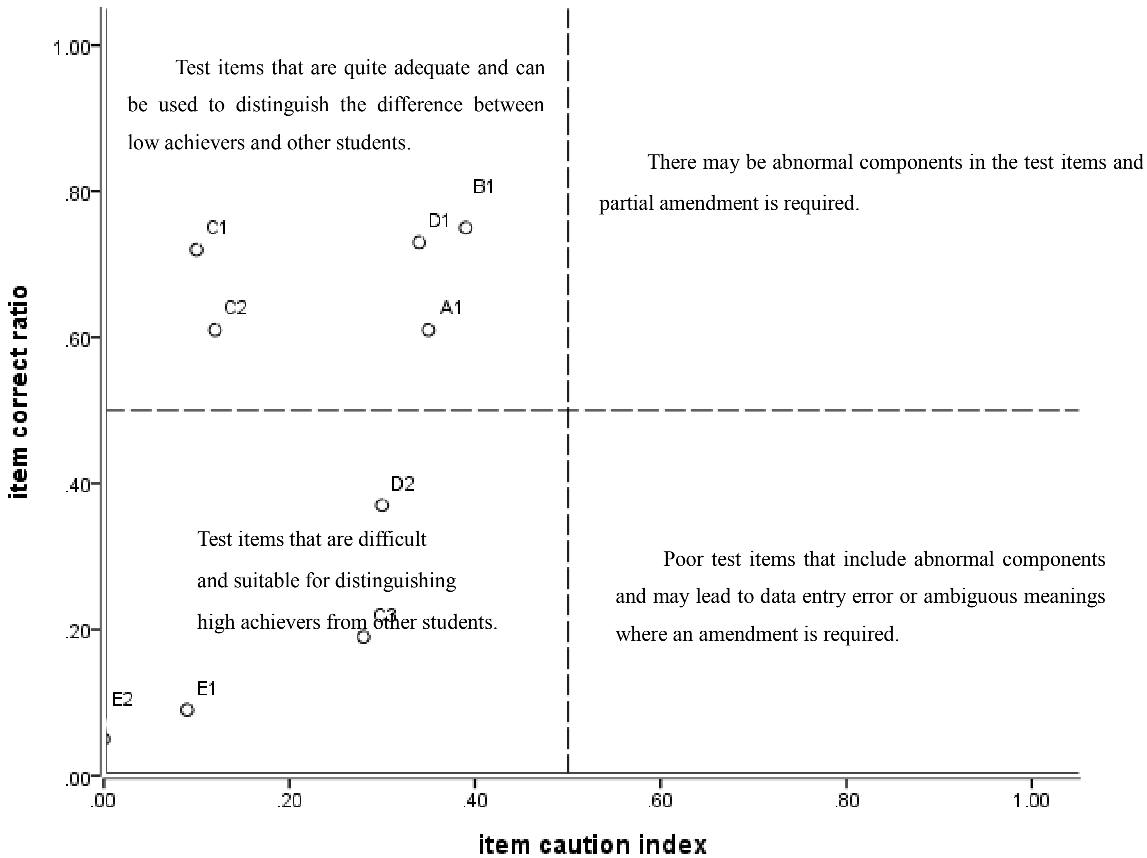


Figure 1. Test item correct answer rate and caution index

Moreover, because the test item of B1 is simple mathematical derivation, it is reasonable that the correct answer rate is high. The answer to the test item of A1 is a law of physics, and such a test item should be the simplest. However, owing to students' carelessness and unfamiliarity with the concepts, the correct answer rate is not as high as it ought to be. The correct answer rate of the test item of A1 should be the highest. However, Figure 1 shows that the correct answer rate of many test items is higher than that of A1. The reason might be that students mainly answered the question by memorizing concepts, instead of using logical inference. Therefore, the mistake made in A1 would be corrected in B1. The distribution of correct answer rate of other items is reasonable. The correct answer rate of the test item of B1 is higher than that of all the test items of C, D, and E. The correct answer rate of various test items of C, D, and E are placed in order according to the level. The test item of E further discusses the phenomenon that quantum mechanics is more inclined to classical physics and involves higher integrated thinking. Therefore, the correct answer rate is undoubtedly the lowest. Based on the above, the hierarchical design of the test items generally conforms to order theory (Lin, 2005).

### *3.2 Students' Problem Solving Performance and Patterns*

Table 2 shows the common two-way classification of students' correct answer rate and caution index (Yu, 1996). The correct answer rate is divided into three levels based on the values.75 and.50. Caution index is divided into two levels based on the value.50. The results of cross interaction are divided into 6 categories: good learning and high stability; stable learning and studying harder is required; lack of learning ability, insufficient learning and studying harder is required; mistakes are caused by carelessness; occasional carelessness, insufficient preparation, and studying harder is required; extremely unstable learning, arbitrary studying habit, and insufficient preparation for test content. The students' learning performance in this study includes all these six categories. Most of their performance is "lack of learning ability, insufficient learning and studying harder is required," followed by "stable learning and studying harder is required."

Table 2. Students' Performance Results Assessment Summary Table

| Students' performance (two-way classification cross interaction of student and caution index) | Scope of correct answer rate | Scope of student caution index | Number of students | Percentage |
|---|---|---|---|---|
| Lack of learning ability, insufficient learning and studying harder is required | 0-.50 | 0-.50 | 35 | 44.3 |
| Stable learning and studying harder is required | .50-.75 | 0-.50 | 31 | 39.2 |
| Good learning and high stability | .75-1.00 | 0-.50 | 3 | 3.8 |
| Extremely unstable learning, arbitrary studying habit, and insufficient preparation for test content | 0-.50 | .50-1 | 4 | 5.1 |
| Occasional carelessness, insufficient preparation, and studying harder is required | .50-.75 | .50-1 | 3 | 3.8 |
| Mistakes are caused by carelessness | .75-1.00 | .50-1 | 3 | 3.8 |
| Total | | .50-1 | 79 | 100 |

The comparison between these students' problem solving performance and all the 4-year college academic performance found that, as shown in Table 2, the correct answer rate of the students with low caution index is generally consistent with their average academic performance. For example, as shown in Table 2, three students with the best academic performance are classified into "Good learning and high stability." Two of them are the best two students with the highest graduation academic performance among approximately 80 graduates in the year. The academic performance of the other one is good as well. However, there are some differences in the test items of high caution index. For example, among 4 students with low correct answer rate and higher caution index, the academic performance of two of them is very good. However, that of the other two is poor. High caution index cannot be used to predict the overall learning performance of students.

### *3.3 Correlation between Students' Problem Soling Performance and Learning Performance*

This study analyzed the correlation between students' problem solving performance for the example and their academic performance to verify that the correlation coefficient was.68 ($p<.001$). The result showed that, students' performance for the example could explain 46.24% of the variance of their academic performance, suggesting that the example is an important starting example for the learning of quantum mechanics.

## 4. Conclusions and Suggestions

The purposes of this study are: 1) to test the correlation between students' learning performance of the course of modern physics and that in the example of "one-dimensional infinite square well"; 2) to investigate the problem solving performance of physics students for "one-dimensional infinite square well" in order to better understand their learning situation and difficulties in this example. The conclusions and suggestion are described as follows:

There are several representative examples in all the main physics courses. If all the instructors can identify them and further provide students with instruction in detail in order to enable them extending their knowledge from the fundamental concepts to high-level thinking, it is possible to help them learn physics courses accurately in more efficient way.

The example in this study is indeed representative. The correlation coefficient between students' performance for this example and academic performance is.68 ($p<.001$).

The S-P chart analysis showed that the quality of test items is good, and there is no bad text item. The analysis result showed that the quality is: Test items that are quite adequate and can be used to distinguish the difference between low achievers and other students. The test items of higher concepts are: Test items that are difficult and suitable for distinguishing high achievers from other students.

Based on the SP chart analysis, the students' performance for this example is mainly "lack of learning ability, insufficient learning and studying harder is required," followed by "stable learning and studying harder is required."

In terms of research restrictions, because the source of samples is one school only, the admission performance of the students is approximately PR 50, and their learning motivation is not high. There is a need to take into account the heterogeneity of samples and environment and carefully perform the deduction.

In terms of research methods, quantitative interviews can be used to probe into the potential causes affecting students with different performance patterns.

Several examples playing the important role as that of "one-dimensional infinite square well system" can be found in main courses of department of physics. If students can be motivated to further think over these examples and instructors can review and further discuss about them, it is believed that students' overall learning efficiency can certainly be significantly improved.

There should also be many important examples existing in various courses of science and engineering departments. If the idea of this study can be promoted to other departments, students' learning efficiency can be changed to a certain extent.

## References

Ausubel, D. P. (1963) *The Psychology of Meaningful Verbal Learning*. London: Frune & Stratton.

Clement, J. (1983) *A conceptual model discussed by Galileo and used intuitively by physics students*.

Fischler, H. & Lichtfeldt, M. (1992). Modern physics and students' concepts. *International Journal of Science Education, 14*(2), 181-190.

Genter, D.&A. Stevens (eds) *Mental Models* (pp. 99-130). Hillsdale, N.J.:Lawrence Erlbaum Associates.

Gentner, D. & Gentner, D. R. (1983) Flowing water or teeming crows: Mental models of electricity. In Johnson-Laird,

P.(1983) *Mental Models*. Cambridge:Harvard University Press.

Gilbert, J. K. & Boulter, C. J. (1995) Stretching models too far. Paper presented at the annual meeting of the American

Educational Research Association (San Francisco).

Greca, I. & Moreira, M. (2000) Mental models, conceptual models and modeling. *International Journal of Science Education, 22*(1), 1-11.

Ireson, G. (2000) The quantum understanding of pre-university physics students. *Physics Education*, *35*(1), 15-21.

Lin, Y. H.(2005). Order Theory. *Journal of Educational Research, 134*, 142-143. McCloskey, M. (1983) Naïve theories

of motion. In D. Dentner & A. Stevens (Eds.), *Mental Models* (pp. 325-340). Hilsdale, NJ: Lawrence Erlbaum Associates.

McCloskey, M. (1983) Naïve theories of motion. In D. Dentner & A. Stevens (Eds.), *Mental Models* (pp. 299-324). Hilsdale, NJ: Lawrence Erlbaum Associates.

Minstrell, J. (1982) Explaining the "at rest" condition of an object. *The Physics Teacher, 20*, 10-14.

Sato,T.(1980). The S-P chart and the caution index. *NCE Educational InformationBulletin*, 80-1.

diSessa, A. (1993) Toward an epistemology of physics. *Cognition and Instruction,10*(2 & 3), 105-225.

Smith, J., diSessa, A. & Roschelle, J. (1993) Misconceptions reconceived: a constructivist analysis of knowledge in transition. *The Journal of the Learning Science, 3*(2), 115-163.

You, S. C., Yu, M.N.(2006). A Study on the Correlation between Knowledge Structure Diagnostic Assessment and S-P Chart. *Educational and Psychological Study, 29*(1), 183-208.

Yu, M. N.(1996). *Principles for the Development of Achievement Test*. Taipei: Hsin Li.