Mathematics Learning Difficulties: An Australasian Perspective

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Abstract: *Although students in Australasia have generally achieved close to international averages in mathematics over the past decade, there have been growing concerns about apparent disparities in achievement, especially for those with learning difficulties. Without strong mathematics skills, the education and career choices of these students will be limited particularly in STEM disciplines. However, the relevant literature to support students with mathematics learning difficulties (MLD), is generated across three research domains: Mathematics Education, Learning Difficulties and Special Education. As a result, to support these students, educators must read across disciplinary boundaries and overcome challenges raised by differing perspectives, concepts and understandings. This paper draws upon a review of recent (2008-2012) Australasian literature across these three domains and proposes steps that need to be taken if educators are to be informed about effective teaching and learning practices for supporting students with MLD.*
Keywords: learning difficulties, special education, mathematics education, low attaining

1. Introduction

Over the last decade, Australian students overall have generally achieved close to international averages in mathematics. Nonetheless, results from the Trends in Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) indicate relatively high disparities in mathematics achievement for diverse populations including those with learning difficulties (Mulligan, 2011). Indeed, 30% or more of Australian students in Years 4 and 8 achieved at or below the lowest mathematics benchmark (Thomson, 2010). Of further concern is that the achievement gap between students with learning difficulties (LD) and their typically developing peers widens as they progress through school (Bellert, 2009). If these underachieving students fail to develop proficiency in mathematics, their preparedness for science, technology and engineering subjects will be limited.

Given these concerns, it is imperative that all classroom teachers – from the early years to senior secondary – utilise evidence-based approaches to teaching and learning in mathematics, particularly to support those with mathematics learning difficulties (MLD). For two reasons, this is not a simple task. First, despite years of national commitment to evidence-based policies and curriculum initiatives in Australia and New Zealand to ensure that all students have access to a high quality education, there is little evidence to date that these endeavours have had a significant positive impact on the mathematics achievement of students considered 'low-achievers' or 'at risk' (Hoad, Munro, Pearn, Rowe, & Rowe, 2005). Second, in order to gain a deep understanding of the key issues and converging evidence for best practices in supporting students with MLD, educators who wish to undertake research on MLD must synthesise evidence from varied perspectives across Mathematics Education (ME), Learning Difficulties (LD) and Special Education (SE). The purpose of this paper is to address this second challenge by summarising findings from a review of current literature informing discourse about students with MLD and proposing steps that need to be taken in order for MLD to be better understood.

2. Review of the Literature

The review of recent MLD literature (see also Diezmann, Stevenson, & Fox., 2012) was completed in two phases. In the first phase, Informit, ERIC and PsycINFO databases were searched using a set of 11 descriptors (e.g., 'mathematics education' and 'learning difficulties'). Manual searches were also conducted of key reference materials (e.g., national and international journals, conference proceedings, thesis databases and publications related to the most recent TIMMS and

PISA assessments (e.g., Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2011). The literature was then categorised into one of three fields (ME, SE, LD) according to the authors' academic background and publications, a methodology previously employed by Porter and Lacey (2005). In the second phase of the review, a thematic analysis of the content and findings of the research literature across the three fields of interest was conducted in order to identify key issues and converging evidence for best practices in supporting students with MLD.

3. Categorisation of MLD Literature

The process of categorising MLD literature according to one particular field was not entirely straightforward in that with several authors affiliated with more than one field while others engaged in collaborative research with academic peers in other fields. Nonetheless, of the total of 55 publications identified, 51 could be categorised within a single category. The literature reviewed ranged in format from peer-reviewed journal articles (36.4%) and conference papers (32.7%), books (5.5%) and book chapters (14.5%), dissertations (7.3%) to research reports (3.6%).

The review revealed that most of the Australasian authors were affiliated with the field of ME (55.1%), followed by SE (16.3%), Other (Early Childhood, Educational Assessment and Psychology) (16.3%) and LD (12.3%). While most publications (39.3%) contained references from all three fields (with nearly half of the papers published in ME journals), other publications contained either references from two fields (also 39.3%), primarily ME and SE or from a single field (21.4%) again, primarily ME.

The review also identified the range of perspectives across the three fields. Within ME, research draws predominately on psychology to establish the causes of MLD in relation to cognitive and processing problems. Much of this literature concentrated on cognitive theory-based interventions focused on developing conceptual understandings (Bellert, 2009; Graham & Pegg, 2008) or promoting fundamental mathematical thinking grounded upon constructivist learning principles (Gervasoni, 2008; Mulligan, 2011). There was little evidence of the eclectic use of different theoretical models and methodologies to investigate MLD or to develop interventions. In comparison, literature in the field of LD focused more broadly on learning problems to do with numeracy – that is, functional aspects of mathematics in contexts of everyday life (Elkins, 2005) – rather than on mathematics. An analysis of references indicated that a number of authors in this field have taken a multi-perspective approach (Gunn & Wyatt-Smith, 2011), utilising findings from a range of theoretical orientations and research methodologies to make evidence-based claims about the effectiveness of certain interventions. Only a small number of MLD studies were identified from the field of SE. Compared with research from ME or LD, the models and methodologies used to investigate MLD within SE were narrower in range, primarily taking a psycho-medical perspective with a focus on the diagnosis of learners' cognitive 'deficits'. Nonetheless, recent SE publications were frequently cited by researchers working in the other two fields.

4. Key Issues in MLD

As noted earlier, the second phase of the review involved a thematic analysis of the content and findings of the identified and categorized literature in order to identify key issues and converging evidence for best practices in supporting students with MLD. The review indicated that there were three key issues. First, there was ambiguity in relation to defining and identifying MLD. Second, there was little consensus about the causes of MLD as well as a lack of interdisciplinary perspective. Third, few studies examined the efficacy of effective intervention strategies beyond a focus on numeracy.

4.1. Issue 1: Defining and Identifying Learning Difficulties

Although a specific 'learning disability' (e.g., in reading or mathematics) has long been recognised in North America, this category of learning need was only recognised in New Zealand in 2007 (Liberty, 2009) and remains a contentious entity in Australia (Gunn & Wyatt-Smith, 2011). As a result, there is no clear distinction between what constitutes a

'learning difficulty' and a 'learning disability' in Australia (Elkins, 2007). This lack of definitional clarity makes comparing, aggregating and synthesising the results of studies problematic (Gunn, 2007).

Comparing and synthesising results across studies is also made problematic by a lack of consensus about the possible causes of MLD as well as how MLD should be identified. As noted by Mulligan (2011), "the research basis for establishing root causes of mathematics learning difficulties lacks the necessary scope and depth and interdisciplinary perspective that may be essential for establishing consensus about research direction and application" (p. 20). Further, likely due to different root causes of their learning difficulties, students will likely vary in their responses to particular supportive learning environments (Gunn & Wyatt-Smith, 2011). Thus, the challenge for researchers is to look beyond a single perspective of MLD and search for ways to capitalize on findings yielded by research from multiple perspectives and domains.

4.2. Issue 2: Limited Interdisciplinary Work on MLD

Contemporary reviews of the literature indicate that there has been considerable change in the theoretical models of LD informing the MLD research base (Gervasoni & Lindenskov, 2011). Much of the change relates to the emergence of new perspectives and paradigm shifts. Westwood (2008) refers to Twomey's (2006) work in suggesting three contemporary perspectives of LD: (1) the deficit model, (b) the inefficient learner model, and (3) the environmental factors model (see Table 1).

Table 1. Models and causes of learning difficulties.

Model	Causes
Deficit	below average intelligence, lack of automaticity in basic academic skills, attentional difficulties, motivational difficulties, visual and auditory processing difficulties, poor memory capacity, language and literacy difficulties, dyscalculia
Inefficient learner	inaccurate or inefficient cognitive strategies, slow and error-prone retrieval of content, non-awareness of intelligent leaner behaviours
Environmental factors	insufficient or inappropriate instruction, little or no differentiation of learning activities, inappropriate assessment tasks

Within the LD and SE fields in particular, the dominant discourse has focused on a 'deficit' model of LD (Yu & Murik, 2008). Current theories and models have largely built upon this medical/biological base and learning difficulties have been attributed to academic, cognitive, motivational and/or neurological deficiencies within the individual learner (Gunn, 2007; Twomey, 2006).

Within the field of ME, an alternative perspective to the deficit model has emerged in the form of the psychological-processing model. From this perspective, MLD has been examined "in terms of inefficient cognitive habits or ways of thinking that may be amendable to change" (Knight, Bellert, & Graham, 2008, p. 173). As a result, researchers have developed and implemented intervention programs which target inefficiencies such as finger-counting to work out number facts (Bellert, 2009; Graham & Pegg, 2010) as well as limitations in problem-solving ability (Mulligan, Mitchelmore, English, & Robertson, 2010). Some researchers in SE have also focused on diagnosing cognitive inefficiencies (Howell & Kemp, 2009).

The third model considers environmental factors as probable causes of MLD (Westwood, 2008) and current research from this perspective has focused on the relationship between underachievement in mathematics and the quality of teaching and the school curriculum (Gervasoni & Lindenskov, 2011). This perspective considers mathematical difficulties to be the result of ineffective teaching practices, particularly during the early years (Westwood, 2008). Recent research has focused in particular on examining the role of teachers' pedagogical content knowledge (PCK) in relation to effective instruction and student achievement (Beswick, 2007/2008).

Although multiple perspectives inform the MLD evidentiary base, the review indicated that most researchers adopt the dominant theoretical orientations and research methodologies in their respective fields to understand the heterogeneous causes of LD and to develop instruction to address underachievement. That is, researchers reporting from the SE and LD fields are disposed towards working from a psycho-medical model of disability to address deficiencies in learners whereas researchers from ME primarily upon a psychological model of LD as cognitive inefficiencies. This difference between deficiencies and inefficiencies can result in a lack of shared understanding among researchers about what competencies, concepts and tasks are used to assess what constitutes mathematical proficiency and to provide adequate support to develop proficiency.

4.3. Issue 3: Effective Strategies to Support Students with MLD

The review demonstrated that Australasian research has focused on the beneficial effects of a wide range of specific teaching and learning practices for students with LD in mathematics or numeracy (Gervasoni & Lindenskov, 2011; Graham & Pegg, 2010). Nonetheless, there remain gaps in the current MLD literature which pose challenges for the establishment of evidence-based practice.

To date, most empirically-based studies focusing on the support of students with MLD has been conducted by researchers within ME and focused on effective early identification and intervention practices (Gervasoni, 2008; Mulligan, 2011; Wright, 2008) such as Mathematics Recovery (Wright, 2008); Numeracy Intervention Research Project (NIRP) (Ellemor-Collins & Wright (2011); and Extended Mathematical Understanding (EMU) program (Gervasoni, 2008). In addition to research on students' current difficulties Mulligan et al. (2010) have also focused on identifying future predictors of learning difficulties by assessing students' knowledge of mathematical pattern and structure – Awareness of Mathematical Pattern and Structure (AMPS) – and designing an appropriate intervention, the Pattern and Structure Mathematics Awareness Program (PASMAP) program.

In comparison to the breadth of MLD research in ME, SE researchers have focused primarily on the diagnosis and assessment of difficulties in number sense as a predictor of learning problems (Howell & Kemp, 2009, 2010). The limited scope of study maybe due to the perceived importance of Number in mathematics or because researchers perceive specialist mathematical knowledge is required to investigate more complex mathematical topics such as Algebra.

The above discussion highlights the need for researchers to consult empirical evidence beyond their respective fields in order to gain detailed insights into the complex nature and heterogeneous causes of LD in mathematics and design appropriate instruction. MLDs involve more than learning problems with Number and numeracy. For researchers to contribute relevant insights, attempts to points of convergence are needed across all conceptual strands of mathematics.

5. Conclusion

Over the past four years, Australasian researchers have focused on MLD from a range of perspectives including the medical model of disability (deficiencies), the cognitive-based inefficient learner model (inefficiencies) and the environmental factors model (ineffective teaching). What is known is that without early identification and intervention, low-attaining students are at risk of longer term underachievement in mathematics. Although there was a lack of consensus about the causes of underachievement and what constituted effective instruction, researchers across the three fields agree that education systems face challenges in ensuring diverse populations of learners become mathematically proficient. We propose three steps that are of critical importance to develop a better understanding of MLD and improve educational provisions through evidence-based practice.

The first step is to develop a consistent terminology about students with MLD so researchers and policy-makers can contribute to shared dialogue. Without shared operational definitions for identifying diverse populations of students with MLD, it will continue to be difficult for researchers to communicate, compare and interpret research findings. Researchers in New Zealand have addressed this issue with a clear definition (Liberty, 2009) but researchers in Australia continued to be challenged by inconsistencies in the definition of LD. While the use of alternative generic terms such as

'special education needs' in the Australian Mathematics Curriculum: (ACARA, 2010) go some way in countering concerns about the use of restrictive terminology to describe diverse learners, it does little to allay concerns about identifying the characteristics of diverse learners (van Kraayenoord, 2008).

The second step is for researchers to validate claims of successful diagnosis instruments and interventions for low-attaining students in mathematics on the basis of detailed empirical evidence and to establish the transferability of these programs across different learning contexts. Given the complex and heterogeneous nature of MLD, more detailed attention will need to be given to understanding the particular characteristics of learners and their learning contexts (Elkins & Wyatt-Smith, 2011). Methodologically, researching MLD will require the use of a range of theoretical perspectives and research methodologies to enrich understandings of contexts for mathematics learning.

The final step is to develop a systematic research agenda to address the intransigent problems faced by students with MLD. While there is recent evidence of significant and innovative studies trialling intervention programs or practice-based projects on MLD, at the same time, there has been a noticeable decline relating to reviews conducted on MLD. This situation needs to be addressed, if researchers and policy makers are to provide guidance on effective mathematics instruction and intervention programs.

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