Facilitating Prospective Mathematics Teachers' Development of Problem Solving Knowledge for Teaching

Olive Chapman
University of Calgary
chapman@ucalgary.ca

Abstract: *This paper addresses problem solving, a process that is central to students' learning of STEM. It reports on a study that investigated an inquiry-based, learner-focused, instructional approach to facilitate prospective secondary mathematics teachers' learning of problem solving for teaching with connections to STEM tasks. The approach was effective in expanding and deepening the participants' understanding of problems and problem solving in ways that could support the learning of STEM. Characteristics of instructional practices that are important to form a basis of prospective teachers' learning about problem solving are identified.*

Keywords: Prospective teachers, problem solving, STEM, inquiry, learner-focused

1. Introduction

This paper addresses problem solving, a process that is central to students' learning of STEM. It reports on a study that investigated an inquiry-based, learner-focused, instructional approach to facilitate prospective secondary mathematics teachers' learning of problem solving for teaching with connections to STEM tasks. This study is part of ongoing research on prospective and practicing mathematics teachers' development of problem-solving knowledge for teaching.

2. Perspectives of Problem Solving

Problem solving is central to doing, learning and teaching mathematics (Schoenfeld, 1992). The National Council of Teachers of Mathematics [NCTM] standards (NCTM, 2000) promote it as an integral part of all mathematics learning that should not be an isolated part of the program but should involve all content areas. However, whether or how this perspective of problem solving gets implemented in the classroom will depend on the teacher. If problem solving should be taught to students, then it should be taught to prospective teachers who are likely to enter teacher preparation programs without having been taught it in an explicit way. If it is to form a basis of teaching mathematics, or STEM in general, then prospective teachers should also understand it from a pedagogical perspective. This study contributes to our understanding of how to help prospective teachers to develop relevant knowledge to teach problem solving in a way that students can apply to STEM.

In this study, problem solving is viewed as a way of thinking, learning, and teaching. It is a non-linear process that involves problems/situations that require solutions for which explicit procedures are not known prior to encountering the problems/situations. These problems/situations could arise in a variety of contexts, with STEM being of particular focus here. In general, problem solving is central to scientific thinking (National Research Council, 1996), to the technological thinking (Savage & Sterry 1990), to the engineering thinking (Lumsdaine & Lumsdaine, 2000) and mathematical thinking (Mason, Burton, & Stacey, 1982). Layton (1993) highlights the relationship for science and technology (Table 1).

Table 1. Problem solving processes by Layton (1993, p. 46).

General model for problem solving	**Science process**	**Technology/Design**
Understand the process	Consider the natural	Determine the

	phenomenon	need
Describe the problem	Describe the problem	Describe the need
Consider alternative solutions	Suggest hypotheses	Formulate ideas
Choose one solution	Select one hypotheses	Select one idea
Take action	Experiment	Make product
Evaluate the product	Does result fit hypotheses	Test product

A column for mathematics and engineering could easily be added to this table. For example, Polya's (1957) model commonly used in mathematics consists of: understand the problem, develop a plan, carry out plan, and look back. Dym, Agogino, Ozgureris, Frey, and Leifer (2005) in relation to engineering highlight specific aspects of systems thinking as: recognizing the systems context, reasoning about uncertainty, making estimates, and performing experiments. Lumsdaine and Lumsdaine (2000) highlight reasons and strategies to promote creative problem solving in engineering students, arguing that it is essential to engineering professionals. Problem solving, then, is essential to learning STEM and when appropriately interpreted could significantly transform classrooms in ways that promote critical and creative thinking; exploration, discovery, and knowledge creation; risk taking; and collaboration.

3. The Instructional Approach

This study investigated a version of this instructional approach that includes STEM-oriented tasks as a means to help prospective secondary mathematics teachers to understand problem solving as a way of thinking and learning. The approach is grounded in *inquiry* (e.g., Dewey, 1938; Wells, 1999), and *noticing* (Mason, 2002) as a basis of learning. The inquiry was learner-focused in that the participants examined their own experience of work on themselves, i.e., a form of self-study, and compared it with others and theory as a basis of validation. The approach consisted of three stages of activities.

Stage 1 focused on individual self-reflection on problems and problem solving. The participants responded to a set of questions and prompts that included: What is a problem? Choose a grade and make a problem that would be a problem for those students. What did you think of to make the problem? Why is it a problem? Is it a 'good' problem? Why? What process do you go through when you solve a problem? If possible, represent the process with a flowchart.

Stage 2 consisted of self-inquiry activities intended to extend the participants' initial conceptions and knowledge. These activities included: (1) Comparing different types of "problems" without solving them in order to explore the nature and goals of problems. (2) Writing and unpacking narratives of their experiences to examine the cognitive and affective components of the behaviors involved in problem solving. (3) Investigating others (e.g., peers, secondary school students) solving problems. (4) Solving STEM tasks within the context of mathematics. These tasks included: (a) Build the tallest free-standing structure with the materials provided [paper, sticky tape, and scissors]; (b) Be a paper-product engineer; (c) Use a graphing calculator to determine behavior of bouncing ball; and (d) Design a parking lot. (5) Developing a model for problem solving, representing it as a flowchart, and applying it to evaluate it.

Stage 3 included activities that required the participants to compare their post-Stage 2 thinking with their pre-Stage 2 thinking; compare their understanding of problems to theory; compare their problem-solving models and flowcharts with those from theory provided to them; and apply their knowledge to critique a current secondary mathematics textbook approved for use in the Province.

Group reflection: Each of the three stages required small group and whole-class interactions.

4. Research Method

The 20 participants were in the second semester of their 2-year post-degree education program. This was their first course in mathematics education, so they had no instruction or theory on problem solving prior to this experience. The data consisted of the participants' written work for all of the activities. There were also field notes and transcripts of their group and whole-class discussions. The analysis began with open-ended coding of the data. The researcher and research assistants, working independently, coded the data. Coding included identifying significant statements about the participants' thinking/knowledge of problems and problem solving and the changes in thinking/knowledge following the stage 2 activities.

5. Findings

The approach was effective in expanding and deepening the participants' understanding of problems and problem solving for teaching. Their thinking of problems shifted from predominantly traditional algorithmic exercises or word problems to an understanding of characteristics that constitute worthwhile problems that supported various levels of thinking and connections within and outside mathematics. Examples of their descriptors of worthwhile problems include: can be many different forms and types, require thinking at a higher level, have one or more solutions and many approaches, make you think in a different way, interesting, not memorization, challenging but solvable for grade level, connect to real-life situations, require more than one attempt to find a solution, open-ended, missing information, process oriented, can involve working with others, can be fun but also stressful, and real world application.

The participants' knowledge of the problem-solving process was also enhanced. For example, their flowcharts of the process shifted from a linear path in which one reads the problem and knows how to get the answer to one that involved moving back and forth in a cycle of being stuck and getting out of stuck to get to a solution. Example of their descriptors of the process include: visualization and verbalization, a life skill, can extend to other things (outside math), depends on solver, needs persistence, relies on previous knowledge, investigation, trying different strategies, using different tools, depends on problem situation, don't know what outcome will be, take your chances and see what happens, way of learning.

In general, the inquiry activities of Stage 2 of the approach allowed the participants to construct knowledge compatible with formal theory of problem solving in STEM. This allowed them to relate to the theoretical approaches they examined in Stage 3 of the approach in a more meaningful way.

6. Conclusion

Overall, the study suggests that engaging prospective teachers in this self-inquiry approach can help them to deepen their understanding and knowledge of problem solving for teaching. This approach allowed them to develop understanding of problems, problem solver, and problem-solving process which could help their students to learn skills necessary for their learning of STEM. Characteristics of instructional practices identified as important to form a basis of prospective teachers' learning about problem solving include: exploring self and others as problem solver; exploring nature/structure of problems; solving challenging problems of STEM contexts individually and in small groups; posing problems; comparing self with others; and creating own model for problem solving. This suggests that if teachers are to hold knowledge to help them to teach about and through problem solving, they should be provided with experiences not only in solving problems, but also with all of these characteristics.

References

Dewey, J. (1938). Experience and education. New York: Simon and Schuster.

Dym, C. L., Agogino, A. M., Ozgureris, O., Frey, D. D., & Leifer, L. J. (2005). Engineering Design Thinking, Teaching, and Learning. Journal of Engineering Education, 103-120.

Layton, D. (1993). Technology's Challenge to Science Education. Buckingham: Open University Press.

Lumsdaine, E., & Lumsdaine M. (2000). Creative problem solving: Thinking skills for a changing world, New York: McGraw-Hill.

Mason, J. (2002). Researching your own practice: The discipline of noticing. New York: Routledge.

Mason, J., Burton, L., & Stacey, K. (1982). Thinking mathematically. New York: Addison-Wesley.

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (1996). The National Science Education Standards. Washington DC: National Academy Press.

Polya, G. (1957). How to solve it (2nd ed.). New York: Doubleday.

Savage, E. & Sterry, L. (1990). A Conceptual Framework for Technology Education. Reston, VA: International Technology Education Association.

Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 334-370). New York: Macmillan.

Wells, G. (1999). Dialogic Inquiry: Toward a Sociocultural Practice and Theory of Education. Cambridge: Cambridge University Press.