

## Learning Mathematics Collaboratively or Individually

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**Abstract:** Teachers commonly allocate students into small groups to learn collaboratively or facilitate them to learn individually. Worked examples are shown to students in order to assist their understanding. Cognitive Load Theory (CLT) is an instructional design theory that generates instructional procedures using the knowledge of human cognitive architecture. It has been used to generate the worked example effect. When students lack relevant knowledge to solve problems, studying worked examples first before practicing problem solving is very effective. Three previous experiments indicated that studying worked examples was just as effective when working individually as when working in groups on problems for which learners had a similar knowledge base. However, the current experiment showed that an advantage for group learning might be obtained if different learners have a different knowledge base that they must share. If complex problem solving involving more than two specific domains of knowledge is used during learning with heterogeneous grouping in terms of acquired specific domain knowledge, a considerable advantage of group learning might be obtained. Possible effective instructions based on CLT for desirable group learning and implications for educational practice are discussed.

**Keywords:** worked example, problem solving, mathematics, cognitive load, group learning

### 1. Introduction

Learning mathematics at schools is often facilitated by solving problems. In a conventional mathematics classroom, teachers describe a worked example before students practice problems, individually or in small groups. A worked example provides explicit guidance on how to solve a problem effectively (Atkinson, Derry, Renkl, & Worthan, 2000). To enhance learning, a worked example study may be subsequently followed by practicing a similar problem (Sweller & Cooper, 1985). On the contrary, problem solving based instructions provide minimal or no guidance when learning to solve problems (Kirschner, Sweller, & Clark, 2006). This paper discusses the result of an investigation into whether learning individually or collaboratively is effective when facilitated with worked example or problem solving instructions, using the perspective of Cognitive Load Theory (CLT).

### 2. Cognitive Load Theory

Cognitive load theory (CLT) is an instructional design theory that generates instructional procedures using the knowledge of human cognitive architecture. A basic information processing model of human cognitive architecture consists of sensory, working and long term memory (Shiffrin & Atkinson, 1969). In our cognitive system, working memory holds the information we are currently attending to and long term memory stores knowledge we have previously learned. CLT focuses on the limitations of working memory when processing novel information. As a consequence of those limitations, instructions should be designed to minimize irrelevant cognitive activity in working memory (Kirschner et al., 2006).

Sweller (2007) states that cognitive activity in our brain, such as constructing new knowledge or solving problems, might be described by five principles of information processes. The first is the information store principle. This states that knowledge stored in long term memory determines how we deal with new information. For example, pursuing well structured knowledge enables experts to process a problem more efficiently than novices. The second is the borrowing and reorganizing principle. It is assumed that almost all knowledge we acquire is borrowed directly from others, for

instance, by reading another's writing. An explicit format of information delivery might assist us in understanding the meaning of such information the first time it is read. During understanding, working memory reorganizes information using our prior knowledge and then stores it into long term memory. The third, the randomness as genesis principle, claims that if we lack prior knowledge then working memory would generate the random searching and testing of an effective way of using a general problem solving strategy, such as means ends analysis. Flowing from this is Sweller's fourth principle, called the narrow limits of change. As a consequence of working memory's limitations when processing novel information, the random generate and test procedure makes learning clumsy. The fifth principle is the environment organizing and linking principle that suggests that if a substantial amount of prior knowledge from long term memory could be presented to working memory, it might free the working memory load to (re)construct a considerable amount of new knowledge.

CLT is particularly concerned with the second principle, borrowing and reorganizing. The effect of a worked example, in either visual or auditory formats, is one of the findings related to this second principle (Atkinson et al., 2000; Kalyuga, Chandler, Sweller, & Tuovinen, 2001). The effect indicates that learning pairs of a worked example and a practice problem facilitates learning better than learning by problem solving only. A worked example is an explicit instruction that allows new learners to grasp new information efficiently. The subsequent problem solving practice would enable them to rehearse immediately the acquired knowledge, thus enhancing learning. Without explicit information, it might not be advantageous to learn by problem solving. This is because a lack of prior knowledge leads new learners to generate means ends analysis which is irrelevant to learning (Ayres & Sweller, 1990).

Notably, worked examples should be designed to minimize extraneous cognitive load. The split attention and redundancy effects need to be considered when designing a worked example. A split attention effect occurs when two or more sources of information, required to understand material, are presented separately and thus have to be mentally integrated. To avoid this, an integration format is suggested (Tarmizi & Sweller, 1988). If each source of information can be understood in isolation but is presented to learners in an integrated format, a redundancy effect that overloads working memory might occur (Chandler & Sweller, 1991). Paas and Van Merriënboer (1994) investigated a variability effect in a worked example study and Van Gog, Paas, and Van Merriënboer (2004) showed process and product oriented worked examples that could be utilized to avoid this problem.

In collaborative learning, students are placed in groups to share knowledge in order to achieve a depth of understanding during learning (Cohen, 1994). Such learning encourages school students to develop cooperative skills (Johnson & Johnson, 1994) which might be useful for social interaction in the community or workplaces. Geary (2007) categorized social interaction as biologically primary knowledge that people may evolve unconsciously during daily life. Subject matter at school might involve domain specific knowledge, such as algorithms to solve mathematical problems, and Geary (2007) categorizes this as biologically secondary knowledge that requires cognitive allocation and so must be explicitly taught. Paas and Sweller (2012) indicated that biologically primary knowledge might be used to facilitate learning biologically secondary knowledge. An example of this would be allowing students to use their knowledge of engaging in group discussions while studying a topic in mathematics. Learning by collaboration with others is considered advantageous since the interaction might assist students in comprehending the learning material or in developing various strategies for problem solving (Schmidt, Loyens, Van Gog, & Paas, 2007). Furthermore, it could be predicted that when specific knowledge requires learning, interactions with other students who possess this knowledge would be a beneficial resource for the learning student.

### **3. Collaborative versus Individual Learning**

A series of four experiments in Indonesian mathematics classrooms investigated the effectiveness of learning collaboratively or individually with worked examples or problem solving instructions. The learning material was developed in accordance with the National Curriculum and the level of prior knowledge of participating Year 7 students, and was not a replication. Each experiment consisted of learning and testing phases. During the learning phase, each

student received a booklet, either based on worked example or problem solving instructions and also either learning individually or collaboratively. Then, they completed two types of tests individually. A ‘similar’ test consisted of problem statement comparable to those in the learning phase, while a ‘transfer’ test consisted of problems with different representations.

The first experiment showed a replication of previous findings in a Geometry study (Retnowati, Ayres, & Sweller, 2010), which was that a worked example instruction significantly facilitated students during learning, either individually or collaboratively, for two different Arithmetic problem types. Moreover, the results of the ‘similar’ test of one of the problem types revealed that learning individually was significantly better than collaboratively (Retnowati, 2011, September-b).

An element interactivity analysis (Sweller & Chandler, 1994) was then used to create two types of problems that were different in their complexity. The second experiment showed that studying worked examples individually was significantly more beneficial than studying worked example collaboratively, particularly for the more complex material during the transfer test. The presentation of other students while learning more-complex worked examples seemed to be extraneous load (Retnowati, 2011, September-a).

The third experiment repeatedly revealed that studying worked examples individually was significantly better than collaboratively. On the contrary, it was found that, particularly for less complex material, problem solving collaboratively was better than individually. It seemed, again, that the presentations of other students while learning worked examples for more complex material was disadvantageous. When presented with less complex problem solving, conversely, the presentation of other students could be advantageous (Retnowati, 2012, August).

It was noted, however, that the collaborative group consisted of students who had acquired the same level of knowledge. The fourth experiment was designed to investigate whether an advantage for group learning might be obtained if different students have a different knowledge base that they must share in the group. Like the group composition in the previous experiments, the homogeneous groups consisted of students who had the same knowledge base and were encouraged to share this during solving problems; whereas the heterogeneous groups consisted of students with a different knowledge base that they must also share to solve problems that require combinations of knowledge.

Parallel to the previous results, the test results indicated that when a student has already acquired all prior knowledge, solving problems individually was significantly better than in the homogenous groups. However in the heterogeneous group, although a group member did not possess all prior knowledge, the other group members would share with them and vice versa. As indicated by the ‘similar’ test results, the students who learned in the heterogeneous groups were significantly better than the equivalent students who learned individually. Without the presentations of others, the individual problem solver who lacked prior knowledge was deprived of a chance to learn.

#### **4. Conclusions**

The research showed that learning can occur when students are allocated to a collaborative group depending on the complexity of the material, the instructions and the composition of the group. Confirming the CLT based research, when learning new material, the study showed that using worked example instructions is more effective than problem solving instructions. Including more complex learning material in worked examples might be particularly advantageous for individual learners. Problem solving instructions involving less complex material could be learned collaboratively in a homogenous group instead of individually. On the other hand, when the problem solving contains complex material, learning individually could be more effective than learning in a homogenous group. Notably, the reverse effect was found for the heterogeneous group.

#### **5. Implications for Educational Practice**

The different effects found when learning complex problem solving in homogeneous or heterogeneous groups in terms of acquired knowledge could have consequences when used in school. In regular classrooms, the school curriculum commonly structures students to learn the same knowledge together in stages. The findings of this study indicate that when group learning consists of students who have acquired the same knowledge base (homogenous), solving problems collaboratively may not be advantageous. Learning individually, in this circumstance, could facilitate greater knowledge acquisition. However, when each group member has acquired a different knowledge base and combinations of knowledge are required to solve a problem, collaborative learning (heterogeneous) could be an advantage. If group learning is desirable in school, then teachers need to structure the curriculum to permit each student to acquire a different knowledge base before instructing them in collaborative work.

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